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Doubly Discordant SH_0ES NGC 4258 Cepheid Relations (HVI) and Impactful Extinction Laws

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Abstract—SH_0ES (Supernovae and H_0 for the Equation of State of dark energy) 2016–2022 HVI data for classical Cepheids in the keystone galaxy NGC 4258 yield doubly discordant Wesenheit Leavitt functions: $\Delta W_{0,H-VI} = -0\overset{\mathrm{m}}{.}13 \pm 0\overset{\mathrm{m}}{.}02$ ($-0\overset{\mathrm{m}}{.}17$ unweighted) and that is paired with the previously noted $\Delta W_{0,I-VI} \simeq -0\overset{\mathrm{m}}{.}3$, which in concert with complimentary evidence suggest the 2016 SH_0ES NGC 4258-anchored $H_0 \pm \sigma_{H_0}$ warrants scrutiny (i.e., $\sigma_{H_0}/H_0 \gtrsim 6\%$). Cepheid distance uncertainties are further exacerbated by extinction law ambiguities endemic to such Leavitt relations (e.g., NGC 4258), particularly for comparatively obscured variables (e.g., $\Delta d \gtrsim 4\%$, reddened Cepheid subsamples in the Milky Way, M 31, NGC 2442, NGC 4424, NGC 5643, NGC 7250). Lastly, during the analysis it was identified that the 2022 SH_0ES database relays incorrect SMC Cepheid photometry.

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1. INTRODUCTION

The keystone galaxy NGC 4258 (M 106) poses a long-standing challenge to Cepheid researchers, as evidenced by a history of both mismatched distances and Leavitt functions (e.g., Majaess, 2024b, and references therein). Complications arise partly from degeneracies fostered by notable galactocentric crowding, surface brightness, and chemical abundance trends, and possibly non-standard obscuration parameters. Recently, a substantial $0\overset{\mathrm{m}}{.}3$ offset was highlighted between the Hoffmann et al. (2016, SH_0ES)[1] and Yuan et al. (2022, SH_0ES) Wesenheit W_{I-VI} magnitudes for NGC 4258 Cepheids (Majaess, 2024a), and inhomogeneities extend to Maoz et al. (1999), Newman et al. (2001), Macri et al. (2006), and Fausnaugh et al. (2015). The Yuan et al. (2022, SH_0ES) findings are favored since they employ an enhanced characterization of complicated photometric contamination (see their Section 4), thereby validating existing concerns calling for such scrutiny of prior SH_0ES endeavors (e.g., Majaess, 2020; Efstathiou, 2020). Importantly, the divergent NGC 4258 results are emblematic of broader difficulties inherent to Cepheids (e.g., photometric contamination, standardization, extinction law, and metallicity, Bono et al., 2008; Fausnaugh et al., 2015; Majaess, 2010, 2024b, see also Section 5 of Macri et al., 2001, right panel of Fig. 9 in Yuan et al., 2020, Section 4 of Yuan et al., 2022, and in unison Fig. 1 in Madore and Freedman, 2023 and Fig. 6 in Yuan et al., 2022). Consequently, TRGB and JAGB distances are desirable, especially when associated with uncrowded and less obscured regions (Freedman and Madore, 2023, see also Breuval et al., 2023). Pertinently, Freedman et al. (2024, CCHP[2]) concluded that Cepheid and SH_0ES distances are too proximate relative to their TRGB and JAGB results, and problems exist prior to the extension by SNe to ascertain H_0. A separate view is espoused by Riess et al. (2024, SH_0ES).

Freedman et al. (2024, CCHP) relayed JWST JAGB and TRGB expansion rates of 67.80 and 68.81 km s^{-1} Mpc^{-1}, while Tammann and Reindl (2013) and Riess et al. (2024, SH_0ES) favored 64.1 ± 2.0 and 73.2 ± 0.9 km s^{-1} Mpc^{-1}, respectively. Those findings are presented while being ever mindful of the Sandage and de Vaucouleurs H_0 debate[3], which underscores the relevance of blinding procedures (e.g., Section 4 in Freedman et al., 2024, CCHP). On that broader topic Chaussidon et al. (2024, DESI[4]) advocate, "To avoid confirmation bias ... blinding

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[1] Irrespective of those concerns, Hoffmann et al. (2016, SH_0ES) presented a seminally expansive W_{I-VI} sample of Cepheids beyond the Local Group.

[2] Chicago–Carnegie Hubble Program.

[3] Overbye (1991).

[4] Dark Energy Spectroscopic Instrument.

Table 1. NGC 4258 SH0ES 2016–2022 data

SH_0ES data ref.	W_λ	$\Delta\beta$
Riess et al. (2016) Riess et al. (2022)	$H-VI$	$-0\overset{\mathrm{m}}{.}13 \pm 0\overset{\mathrm{m}}{.}02$
Hoffmann et al. (2016) Yuan et al. (2022)	$I-VI$	$-0\overset{\mathrm{m}}{.}3$

For W_{I-VI} findings, see Majaess (2024a).

procedures become a standard practice in the cosmological analyses of such surveys.” Yet, SH_0ES lacks a published comprehensive blinding process, and the Riess et al. (2005, 2024) H_0 remained comparatively unaltered across nearly two decades despite internal inconsistencies, which challenge the notion of robustness and cumulative convergence. Research by Efstathiou (2020) implies the following: vetting SH_0ES is partly hindered by unpublished pre-culled Cepheid datasets, there are Leavitt slope offsets (e.g., see also Fig. 2 in Majaess, 2010), the uncertainties are underestimated (e.g., see also Table 1 in Majaess, 2024a), and sizable color offsets exist (e.g., see also bottom left panel of Fig. 6 in Majaess, 2010). Certain SH_0ES compilations can be challenging to scrutinize, because their VI photometry is not decoupled into fully decontaminated separate bands, and early SH_0ES data should be interpreted cautiously, since the W_{V-VI} slope in Riess et al. (2009b, SH_0ES) determined for solar-Cepheids contradicted Local Group counterparts (compare Fig. 12 in Riess et al., 2009b to Fig. 2 in Majaess, 2010, and Fig. 1 in Majaess et al., 2011). The shallower slope determined by Riess et al. (2009b, SH_0ES) may be indicative of non-standard photometry or inaccurate decontamination (e.g., Fig. 2 in Majaess, 2020), and furthermore a subset of SH_0ES observations was too blue (e.g., NGC 1309, Majaess, 2010). Moreover, cross-referencing SH_0ES Cepheids solely on the basis of coordinates can prove unsatisfactory (see also Efstathiou 2020, and discussion therein). Issues expressed regarding SH_0ES are conveyed in several studies (e.g., Efstathiou, 2020; Mörtsell et al., 2022; Madore and Freedman, 2023; Blanchard et al., 2024; Freedman et al., 2024; Wojtak and Hjorth, 2024; Gavas et al., 2025; see also Majaess, 2010, 2024a, b).

Concurrently, rational foundational concerns persist relative to ΛCDM (e.g., Steinhardt, 2011; Kroupa et al., 2012; López-Corredoira, 2014; Peebles, 2015)[5], and hence H_0 tied to that model (e.g., see Section 1 and Appendix A in Riess and Breuval, 2023). A summary of earlier-epoch H_0 ΛCDM estimates by the NASA LAMBDA[6] team included:

$71.2 \pm 2.1\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ (SPTPol 2017),
$67.36 \pm 0.54\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ (Planck PR3 2018),
$68.7 \pm 1.3\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ (Planck+ACTPol+SPTPolEE 2021).

The spread partly arises from inhomogeneous coverage, yet expands when considering models beyond canonical ΛCDM. That extended ensuing baseline can be compared to independent relatively local H_0 determinations (Steer, 2020).

In this study, additional issues regarding NGC 4258 data are relayed. In Section 2.1, W_{H-VI} magnitudes tied to SH_0ES 2016–2022 NGC 4258 observations are compared. In Section 2.2, the compounding impact of an uncertain ratio of total-to-selective extinction R on such Wesenheit magnitudes is reassessed, and systematic distance shifts are characterized. R linked to NGC 4258 may be anomalous (Fausnaugh et al., 2015, and caveats therein), and a debate continues regarding the broader ramifications of R-uncertainties (Mörtsell et al., 2022; Wojtak and Hjorth, 2024; and see Riess et al., 2022 for the counterpoint).

2. ANALYSIS

2.1. Photometric Inhomogeneities

A Wesenheit formulated Leavitt Law is:

$$W_{H-VI} - W_{0,H-VI} = \mu_0$$
$$(H - R_{H-VI}(V-I)) - (\alpha \log P + \beta) = \mu_0$$

For example, the filters can be H ($F160W$), V ($F555W$), and I ($F814W$). The selected ratio of total-to-selective extinction ($R_{H-VI} \approx 0.4$) is the midpoint cited by Riess et al. (2016, SH_0ES), who noted, “ranging from 0.3 to 0.5 at H depending on the reddening law” (see also Table 6 in Riess et al., 2022). The impact of that spread is discussed in Section 2.2. The slope (α) stems from an analysis of the Riess et al. (2019, SH_0ES) observations for LMC Cepheids owing to the relative insensitivity of that term to metallicity in the passbands examined (e.g., Riess et al., 2022).

A broader problem is promptly elucidated by comparing zeropoints of the absolute Wesenheit magnitude relation inferred from Riess et al. (2019) SH_0ES LMC Cepheids ($\beta = -2\overset{\mathrm{m}}{.}53 \pm 0\overset{\mathrm{m}}{.}03$), Riess et al. (2016) S$H_0$ES NGC 4258 Cepheids ($\beta = -2\overset{\mathrm{m}}{.}72 \pm 0\overset{\mathrm{m}}{.}04$), and Riess et al. (2022) SH_0ES NGC 4258 Cepheids ($\beta = -2\overset{\mathrm{m}}{.}59 \pm 0\overset{\mathrm{m}}{.}03$). Those results are tied

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[5] Significant issues associated with canonical cosmology include: “... dark matter particle or not, explanation of cosmic acceleration, the transformation of inflation into a fundamental theory.” —UChicago Cosmic Controversies conference.

[6] Legacy Archive for Microwave Background Data Analysis.

to the Pietrzyński et al. (2019) LMC and Reid et al. (2019) NGC 4258 anchor points following SH_0ES. In concert with the Majaess (2024a) findings (their Section 2), the results point to problems endemic to the 2016 SH_0ES VIH NGC 4258-based H_0 and σ_{H_0} (Tables 6 and 8 in Riess et al., 2016)[7]. Alternatively, when the same galaxy is utilized, its distance (& uncertainty) can be obviated and the significance of the offset strengthens, whereby NGC 4258 results inferred from Riess et al. (2016, 2022, SH_0ES) subtract to yield: $\Delta\beta = -0.13 \pm 0\overset{m}{.}02$ or an unweighted $-0\overset{m}{.}17 \pm 0\overset{m}{.}03$ (Table 1). The problems arise owing to an apparent Wesenheit magnitude offset between those NGC 4258 datasets (ΔW_{H-VI}), since there are $\Delta H \Delta(V-I)$ photometric deviations amongst common stars. That is the cause, and the effect can be observed upon β or $W_{0,H-VI}$. That is likewise true of the Hoffmann et al. (2016, SH_0ES) and Yuan et al. (2022, SH_0ES) NGC 4258 discrepancy (see also Section 4 of the latter). A contested metallicity effect does not reconcile all aforementioned data (e.g., contrast Madore and Freedman, 2023; Riess and Breuval, 2023).

2.2. R-Ambiguities

Fausnaugh et al. (2015) posited that NGC 4258 could adhere to an anomalous $R_V \simeq 4.9$ relative to the Milky Way ($R_V \simeq 3.26$, Berdnikov et al., 1996), yet cautioned and preferred that, "it seems probable that at least one other systematic effect is at work in our sample's colours". Furthermore, a galaxy could possess R-variations as a function of galactocentric distance (e.g., Gontcharov, 2013), in tandem with a mean diverging from a broader extragalactic sample (e.g., Gordon et al., 2003). Fitzpatrick (1999) remarked that a "bewildering variety of IR-through-UV extinction curves" are commonplace, and Gordon et al. (2003) noted that the Milky Way and SMC extinction curves exhibit a "continuum of properties" (see also Tables 2 and 4 in Turner, 1976, and Table 1 in Turner, 2012). Specifically, the Galactic Bulge may follow a separate R_{VI} relative to the Solar Neighborhood (e.g., Udalski, 2003), and Majaess et al. (2016) confirmed that Carina features an extreme optical R_V (e.g., Turner, 2012; Carraro et al., 2013), whereas the near-infrared R_{J-JK_s} results were comparatively constant across ℓ. A debate persists whether infrared extinction laws vary (e.g., Zasowski et al., 2009), and if a subset of passbands within that domain are acutely sensitive to compositional changes (e.g., Hackwell and Gehrz, 1974; Scowcroft et al., 2011; Majaess et al., 2013).

As was noted, R_{H-VI} could span 0.3–0.5, while R_{I-VI} may traverse 1.30–1.55, and those estimates are conservative (e.g., Table 2 in Udalski, 2003, and Table 6 in Riess et al., 2022), and change slightly with marginally different SED-filter-instrument profiles. For example, Hoffmann et al. (2016, SH_0ES) adopted $R_{I-VI} = 1.45$, whereas Yuan et al. (2022, SH_0ES) selected $R_{I-VI} = 1.30$ following Riess et al. (2019, SH_0ES, and references therein). Regarding the W_{H-VI} passband combination, Riess et al. (2009, SH_0ES) favored $R_{H-VI} = 0.479$, whereas Riess et al. (2022, SH_0ES) derived $R_{H-VI} = 0.34$ from their full Cepheid sample.

In sum, there exists a breadth in the potential extinction law employed for extragalactic Cepheids. An avenue to approximate the impact on distances by the aforementioned spread is to derive absolute and apparent Wesenheit functions using the same R. First, the Riess et al. (2019, SH_0ES) LMC photometry was utilized to establish the absolute Wesenheit magnitude, in tandem with the Araucaria anchor[8]. Second, distances were subsequently computed for individual Galactic Cepheids by relying on Riess et al. (2021, SH_0ES) for the apparent Wesenheit computation. That SH_0ES Milky Way photometry advantageously samples a sizable extinction baseline. The ensuing median distance offset between R_{I-VI} extrema is about 5%, with lower R_{I-VI} yielding further distances. For R_{H-VI}, an approximately 4% median difference was identified. The largest distances established using $R_{H-VI} = 0.30$, relative to the shortest determined via $R_{I-VI} = 1.55$, were separated by about 8% (median). R_{H-VI} extrema distances were generally more remote than those tied to R_{I-VI}.

The broadly illustrative approach was likewise applied to the expansive extragalactic data of Riess et al. (2016, 2022, SH_0ES) and Hoffmann et al. (2016, SH_0ES), but this time for all $P > 7^{\rm d}$ Cepheids within a given galaxy (i.e., period criterion mitigates overtone contaminants). Cepheids in M 31, NGC 2442, NGC 4424, NGC 5643, and NGC 7250 featured sightlines with comparatively enhanced obscuration, and possessed a more than 4% distance offset. NGC 4258 falls short of that group ($\Delta d \lesssim$ 1.6%, dataset depending), if R lies within the aforementioned bounds and is not anomalous (however, see Fausnaugh et al., 2015, and caveats therein). That uncertainty is paired with the more dominant W_{H-VI} SH_0ES photometric offset (Section 2.1), which implies[9] an approximately 6% H_0 internal

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[7] See also the important findings conveyed in Section 10 of Freedman and Madore (2023).

[8] Such anchors may require revision (importantly, possibly unidirectionally), and separate determinations exist that possess low cited uncertainties (Steer, 2020, and references therein).

[9] Eq. (2.1) in Efstathiou (2020).

discrepancy. The H_0 inconsistency is significantly higher amongst the W_{I-VI} SH_0ES 2016–2022 data (Majaess, 2024a).

Lastly, the macro data inspection indicated that SMC Cepheid W_{H-VI} photometry compiled by Riess et al. (2022, SH_0ES) appears awry, and overlays upon the LMC Wesenheit function. That conclusion was confirmed by constructing a sample of SMC Cepheids by drawing from photometric catalogs in the literature. More broadly, the SMC can pose a comparatively greater challenge as a Wesenheit calibrator (e.g., LMC) granted its sizable depth along the sightline and inclination (Caldwell and Coulson, 1986; Groenewegen, 2000; Subramanian and Subramaniam, 2012, e.g., the latter's Fig. 12).

3. CONCLUSIONS

The absolute $W_{0,H-VI}$ magnitude zeropoint for Cepheids in the critical maser galaxy NGC 4258 spans greater than $0\overset{m}{.}1$ across the SH_0ES 2016–2022 datasets (Table 1). That adds to a $0\overset{m}{.}3$ $W_{0,I-VI}$ discrepancy unveiled previously and linked to a separate passband combination (Majaess, 2024a), and points to problematic 2016 SH_0ES NGC 4258 photometry and $H_0 \pm \sigma_{H_0}$ anchored to that galaxy (see entries in Tables 6 and 8 in Riess et al., 2016). Moreover, ensuing Cepheid distance uncertainties are further enlarged beyond that SH_0ES internal discrepancy (an implied approximate 6% H_0 when associated solely with that maser anchor) for relatively reddened subsamples owing to R_{H-VI} and R_{I-VI} ambiguities (Section 2.2).

Continued independent research on the broader impact of R-variations on Cepheid and SN distances is desirable (e.g., Elias-Rosa et al., 2006; Goobar, 2008; Fausnaugh et al., 2015; Wojtak and Hjorth, 2024), as exemplified by the ambiguity regarding the distance to Centaurus A (Ferrarese et al., 2007; Majaess, 2010). Constraints could emerge from breakthroughs in characterizing the interstellar medium by revealing source(s) behind the 220 nm extinction bump, numerous unidentified infrared emission lines, and more than 500 diffuse interstellar bands (e.g., Turner et al., 2014; Xiang et al., 2017; Ebenbichler et al., 2024; Majaess et al., 2025).

The SH_0ES team conveys their perspective on how R-uncertainties affect H_0 in several studies (e.g., Riess et al., 2009a, 2022, and the latter's Section 6.3 and Appendix D), and for separate opinions see Mörtsell et al. (2022) and Wojtak and Hjorth (2024).

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CONFLICT OF INTEREST

The author of this work declares that he has no conflicts of interest.

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